Labor-Market Frictions and Endogenous Production-Network Formation *

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Abstract

This paper studies the interaction of labor-market frictions and endogenous production-network formation in firms' decisions on employment, sourcing and production, within the context of trade policy. I construct a quantitative general-equilibrium model featuring worker-firm and firm-to-firm matching in the production process. I calibrate the model and investigate how trade policy and labor-market efficiency affect production-network density, trade flows, unemployment and wage inequality. The model shows a 1.5% decrease in relative labor-market frictions in one country can lead to a 0.1% decrease in the partner country's relative number of downstream production-network linkages. I explore how heterogeneity in firm-to-firm matching costs determines the effect of a trade shock on wage inequality. I show that a protectionist tariff by one country leads to increases in domestic long-run unemployment and real wages. Examining these channels separately leads to sizable quantitative and qualitative changes in the predicted outcomes of trade policy.

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1 Introduction

Trade in intermediates accounts for about half of global firm sales, with cross-border trade in production networks playing a key role. Cross border trade in production networks is determined by comparative advantage, which is driven in part by the difference in relative labor market standards across countries. [1] It is important to study the interaction of these mechanisms to understand the impact of trade policy on workers and the structure of the domestic production network. Surprisingly, the interaction of these two channels has not been studied within the context of trade policy.

In this paper, I quantify how labor market outcomes and production-networks change in response to economic shocks, specifically trade shocks. More specifically, this paper asks the following two questions: How do changes in the production-network after a shock, drive changes in labor-market outcomes, such as employment, unemployment, and wage-inequality, across sectors? How does the reallocation of labor across sectors in response to a trade shock, change the production-network and cross-border trade? To investigate these questions I build a model of endogenous production-network formation in which workers and firms face labor-market frictions.

Despite the volume of trade that occurs between firms, the number of upstream and downstream linkages between firms can be small. Among Japanese firms, for example, the typical number of customers and suppliers is both only three, pointing to significant costs that are incurred to establish and maintain production networks.[2] Furthermore, firms produce combining intermediates with labor services. The interaction between firms' intermediate sourcing decisions as they form and dissolve production-network linkages and their hiring decisions is important in understanding how changes in the costs of networking and trade affect hiring and the level of economic activity, inequality between workers, and the structure of the economy inside and outside of production networks.

Models of endogenous production-network formation have assumed that all firms use labor in

[1] See Helpman (2010)

[2] SeeBernard et al. (2019).

production, which they must source from a friction-less competitive market. Allowing for frictions in the sourcing of labor will change how production-networks form and shift in response to trade policy. This change will be different from an adjustment of firm productivities or network centrality. While an increase in the productivity of one sector will benefit other sectors as well (through an increase in the output and lowering of the unit cost of production for firms in all sectors), an increase in the labor-market efficiency of one sector will lead to a reallocation of labor, which will benefit one sector at the expense of the other. The reallocation of labor that occurs in response to a shock implies that network centrality will shift across sectors in response to a shock. The implications for this are critical for understanding how welfare, employment, and wages respond to trade shocks differently across sectors.

Allowing for the endogenous formation of production-networks will also change how labor-market outcomes respond to shocks. Relative to a model that treats the production-network as exogenous, a model featuring endogenous production-network formation will lead to a change in the responsiveness of labor-market outcomes to trade costs. The sign of this relative change depends on the sensitivity of the production-network to trade shocks. If there is a large (small) change in the number of firm relationships in response to a trade shock, then there will also be a relatively large (small) change in labor market outcomes, relative to a model featuring exogenous production-networks.

Prior work on endogenous production-network formation has mostly focused on the role of firm-level productivities and network centrality in determining the density of the production network.[3]. Lim (2018) builds a model where firm-specific production and demand differences drive differences in network centrality. Here I present a new mechanism for driving density differences across production networks, the difference in labor-market frictions across sectors. This paper builds a model that points out that sectors with relatively lower labor-market frictions will be more central in the production-network. This reflects the fact that firms in sectors with relatively lower labor-market frictions are more profitable for other firms to trade with. Firms in sectors with rel-

[3]See Bernard and Moxnes (2018) for a literature review

atively lower labor-market frictions will produce more output (making them more attractive as a potential customer) and operate at lower unit costs (making them more attractive as a potential supplier).

Prior work on the responsiveness of production networks to shocks also have ignored the reallocation of employees across sectors, focusing only on how shocks propagate throughout the economy based on firm-level productivities and the costs of face-to-face interaction. Huneeus (2018) studies how the effects of the great recession propagated across firms in the Chilean economy. In his model, exposure to the shock occurs through the endogenously formed production network, which is determined by firm-level productivities. Bernard et al. (2019) study the effects of a new high speed commuter rail in Japan and find that reducing the costs of face to face interaction increases the thickness of the production network for firms located along the new rail line. While these features are still present in my model, the addition of labor-market frictions induces a new effect. With the introduction of labor-market frictions, some workers will now be reallocated across sectors in response to the shock. This will affect the ability of firms in each sector to form production-network linkages differently. The reallocation of labor across sectors will lead to firms in one sector being able form production-network linkages more easily, and in the other sector it will now be more difficult for firms to form production-network linkages. This suggests that models of production-network formation which do not take into account labor-market frictions misstate the rate at which production networks respond to trade shocks, and thus misstate the rate at which shocks propagate throughout the economy.

My analysis also contributes to the literature that investigates how trade shocks, offshoring, and outsourcing affect labor-market outcomes. [4] In this paper I present the first framework to examine how employees are affected by shocks through changes in firm-level production-networks. Previous papers have considered how labor markets respond when they are exposed to economic shocks through an exogenous production-network, for example, through input-output linkages. However, these models ignore how the production-network itself might shift and thereby change the level

[4] Acemoglu et al. (2014), Autor et al. (2013), Caliendo et al. (2019), Grossman and Helpman (2005), Helpman and Itskhoki (2010)

of exposure to these shocks. Production networks must change in response to economic shocks, affecting the rate at which these shocks propagate throughout the economy. Shocks will change the labor-market outcomes across all sectors in the economy, and the way that shocks propagate across sectors will be determined by the way that production-network linkages respond to the shocks. In this paper I demonstrate how changes in endogenous production networks lead to changes in the labor market that systematically vary across sectors, and by production-network centrality.

Finally, this paper contributes to the literature on how wage inequality responds to trade shocks. Costinot and Vogel (2010), Egger and Kreickemeier (2009), Helpman et al. (2017) and Helpman et al. (2018), all explore how trade between countries drives wage inequality. These papers explore how wage-inequality will change in response to trade shocks, given firms are operating in an environment featuring worker-firm matching, or fair wage conditions. This paper contributes to this literature by exploring how firm-to-firm matching drives changes in within- and across-sector wage-inequality, in response to trade shocks. I show how the level of heterogeneity in firm-to-firm matching costs determines the sign of the response of within-sector wage-inequality to trade shocks. This implies that trade liberalization could theoretically lead to a reduction in within sector wage inequality.

In what follows I first present a model of endogenous production-network formation that includes labor-market frictions. The model demonstrates not only, how endogenous production-network formation affects labor-market outcomes, but also how labor-market frictions determine the production-network centrality of sectors in the economy. In the following section I present the model and discuss the predictions of the model. I then simulate the model in Section 3. I then simulate the effects of globalization, and the effects of the Home country unilaterally imposing a tariff on a particular sector in the foreign country. I then compare the simulation results to other models featuring trade and labor-market frictions. In order to demonstrate the effect of labor-market frictions on endogenous production-network formation, I compare the model presented in Section 2 to an analogous extension that includes an immobile labor supply. To demonstrate the effect of endogenous production-network formation on labor-market outcomes, I simulate an extension to the model where there is an exogenous input-output structure to trade in intermediates.

2 A Double Sided Matching Model of Production

In this section I build a model of endogenous production-network formation with labor-market frictions in which firms must pay a fixed cost to match with each customer firm, and they must also pay a cost to post employment vacancies. The economy consists of two countries and two sectors, each of which has a continuum of firms. Each country has a fixed labor supply of identical workers that optimally searches for employment across sectors. Sectors and countries vary by the average cost of firm-matching, and the cost of posting an employment vacancy. The model consist of a collection of household preferences as well as production and labor-market structures. In what follows, I describe each of these components in detail, discuss the equilibrium conditions of the model, and finally discuss the predictions of the model with respect to within sector and country wage inequality.

2.1 Basic Environment

The economy consists of two countries, Home (H) and Foreign (F), each consisting of two sectors, indexed by s. Within each country-sector pair there is a continuum of firms, each with a firm specific labor augmenting technology parameter, indexed by ϕ_ℓ. Firms are indexed by the triplet $\phi = (\phi_\ell, s, c)$, where $c \in \{H, F\}$ and $s \in \{1, 2\}$. The economy is monopolistically competitive, with each firm producing its own unique variety that it can sell to both households and other firms. Firms produce by obtaining intermediate varieties, produced by other firms, and combining them with the labor that the firm is able to employ. In order to hire workers, firms must post vacancies and must pay a cost per each vacancy posted. The firms cannot costlessly adjust their labor supply which induces workers and firms to engage in wage bargaining and generating wage differences across firms within sectors.

In what follows I present the model by first considering the household's problem and then examining the firm's problem. Next, I define the equilibrium conditions of the model. I then discuss how changing the labor market parameters lead to changes in the formation of each firm's production-network. I discuss how labor market outcomes change depending on the production-network efficiency. Finally I discuss the implication of changes in trade costs and discuss the predictions of the model in response to a protectionist tariff.

2.1.1 The Household's Problem

Within each sector for a given country, (s, c) the representative household supplies $L_{s,c}$ units of homogeneous labor. The household has love of variety preferences given by:

$$U_{s,c} = \left[\sum_{s',c'} \alpha_{s'}^{HH} \int [x_{s,c}^{HH}(\phi')]^{\frac{\sigma^{HH}-1}{\sigma^{HH}}} dF_{\phi_\ell}(\phi'_\ell) \right]^{\frac{\sigma^{HH}}{\sigma^{HH}-1}}$$

where α_s^{HH} governs the degree to which households value goods from sector s, $\sigma^{HH} > 1$ is the elasticity of substitutions between firm specific varieties. Total demand for each firm's brand by the representative consumer in sector country s, c is given by the following:

$$x_{s,c}^{HH}(\phi') = \frac{I_{s,c}}{P_{s,c}} \left(\frac{\alpha_{s'}^{HH} P_{s,c}}{\tau_{s',c'}^{c} p^{HH}(\phi')} \right)^{\sigma^{HH}} \tag{2.1}$$

where

$$P_{s,c} \equiv \left(\sum_{s',c'} (\alpha_{s'}^{HH})^{\sigma^{HH}} \int [\tau_{s',c'}^{c} p^{HH}(\phi')]^{1-\sigma^{HH}} dF_{\phi_\ell}(\phi'_\ell) \right)^{\frac{1}{1-\sigma^{HH}}} \tag{2.2}$$

is the price index of the representative consumer. $I_{s,c}$ is the total income of the household in s, c. The parameter $\tau_{s',c'}^{c}$ is an iceberg trade cost term that determines how costly it is to ship sector-s' goods from country c' to country c. $p^{HH}(\phi)$ is the price charged to households by firm ϕ. Total demand that firm ϕ faces from households in the economy is given by:

$$x^{HH}(\phi) = \sum_{s',c'} x_{s',c'}^{HH}(\phi) \tag{2.3}$$

2.1.2 Firm Production and Firm-to-Firm Matching

Firm Production Each firm produces its own variety of the differentiated product with a CES production technology using intermediates it obtains from other firms and the labor that it hires, given the labor-market frictions. Firm-ϕ's production function is given by:

$$Y(\phi) = \left(\alpha_s \phi_\ell [\ell(\phi)]^{\frac{\sigma_{\ell,s}-1}{\sigma_{\ell,s}}} + (1-\alpha_s)[x(\phi)]^{\frac{\sigma_{\ell,s}-1}{\sigma_{\ell,s}}} \right)^{\frac{\sigma_{\ell,s}}{\sigma_{\ell,s}-1}}$$

where employment is given by the term $\ell(\phi)$. The firm combines intermediates from other sectors via CES aggregator that is denoted by $x(\phi)$, which will be discussed shortly. The parameter governing the input suitability for the industry of the firm, s, is given by α_s. The elasticity of substitution between workers and aggregate intermediates is denoted as $\sigma_{\ell,s}$.

Given the assumption that the firm cannot costlessly adjust its labor supply, the firm only retains a share of the profits that it is able to generate. This share is given by $1/(1+\beta)$, where β is the bargaining power of the workers. Thus the firm only bears this share of the total-costs that it incurs. Given this assumption, the demand for labor and aggregate intermediates are:

$$\begin{aligned} \ell(\phi) &= \alpha_s^{\sigma_{\ell,s}} \phi_\ell^{\sigma_{\ell,s}} [b_{s,c}]^{-\sigma_{\ell,s}} Y(\phi) \eta(\phi)^{\sigma_{\ell,s}} \\ x(\phi) &= (1-\alpha_S)^{\sigma_{\ell,s}} \left[\frac{P(\phi)}{1+\beta} \right]^{-\sigma_{\ell,s}} Y(\phi) \eta(\phi)^{\sigma_{\ell,s}} \end{aligned} \tag{2.4}$$

where the unit cost function of the firm is denoted by $\eta(\phi)$ and is given by:

$$\eta(\phi) \equiv \left(\alpha_s^{\sigma_{\ell,s}} \phi_\ell^{\sigma_{\ell,s}} [b_{s,r}]^{1-\sigma_{\ell,s}} + (1-\alpha_s)^{\sigma_{\ell,s}} \left[\frac{P(\phi)}{1+\beta} \right]^{1-\sigma_{\ell,s}} \right)^{\frac{1}{1-\sigma_{\ell,s}}} \tag{2.5}$$

The cost of hiring one more worker in sector country-s, c is denoted by $b_{s,c}$ and $P(\phi)$ is the marginal cost of $x(\phi)$.

The firm combines intermediates across sectors using the following CES aggregator:

$$x(\phi) = \left(\sum_{s' \in \{1,2\}} \alpha_{s,s'} [x_{s'}(\phi)]^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}}$$

The parameter $\alpha_{s,s'}$ governs the input suitability of sector s' varieties that are used in the production of sector s goods. σ_s is the elasticity of substitution between sectors used in production by sector s. $x_{s'}(\phi)$ is a CES aggregator of the firm's demand for sector s' varieties. The demand for aggregate intermediates from sector-s' by the firm is given by:

$$x_{s'}(\phi) = (\alpha_{s,s'})^{\sigma_s} [P_{s'}(\phi)]^{-\sigma_s} x(\phi) P(\phi)^{\sigma_s} \tag{2.6}$$

The firm's marginal cost of $x(\phi)$ is defined as:

$$P(\phi) \equiv \left(\sum_{s' \in \{1,2\}} (\alpha_{s,s'})^{\sigma_s} [P_{s'}(\phi)]^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}} \tag{2.7}$$

Where $P_{s'}(\phi)$ is the firm's unit cost of increasing it's sector s' CES aggregator by one unit.

Within its demand for inputs from a given sector, the firm aggregates across varieties from each country. However, before the production and input demand function are presented in detail, I layout the framework of the production-network structure.

There are relationship frictions between firms such that firm ϕ' can only sell to firm ϕ if firm ϕ' pays a fixed cost. This will occur with probability $m(\phi, \phi')$, which I refer to as the *firm-to-firm matching function*. For now the *firm-to-firm matching function* is taken as given, but will be solved for later in the model. This function will determine the margins of trade in the model, besides the intensive margin of trade. I assume that the fixed cost of firm-to-firm matching is paid in terms of labor.

Given the matching function, the firm combines intermediate varieties within each sector according to the following CES-aggregator:

$$x_{s'}(\phi) = \left(\sum_{c' \in \{H,F\}} \int m(\phi, \phi')[x(\phi, \phi')]^{\frac{\sigma_{s,s'}-1}{\sigma_{s,s'}}} dF_{\phi_\ell}(\phi'_\ell | s') \right)^{\frac{\sigma_{s,s'}}{\sigma_{s,s'}-1}}$$

where $x(\phi, \phi')$ is the total sector s' variety demanded by the firm and $\sigma_{s,s'}$ is the elasticity of substitution that governs how sector s firms substitute between sector s' varieties. Given this structure, the conditional demand for variety ϕ' by firm ϕ is given by:

$$x(\phi, \phi') = [\tau^c_{c',s'} p(\phi, \phi')]^{-\sigma_{s,s'}} x_{s'}(\phi) P_{s'}(\phi)^{\sigma_{s,s'}} \tag{2.8}$$

where the firm's unit cost of increasing it's sector s' CES aggregator, $P_{s'}(\phi)$, is defined as:

$$P_{s'}(\phi) = \left(\sum_{c' \in \{H,F\}} \int m(\phi, \phi')[\tau^c_{c',s'} p(\phi, \phi')]^{1-\sigma_{s,s'}} dF_{\phi_\ell}(\phi'_\ell | s') \right)^{\frac{1}{1-\sigma_{s,s'}}} \tag{2.9}$$

where $\tau^c_{c',s'}$ is the cost of shipping sector s' goods from country c' to country c and is assumed to take the form:

$$\tau^c_{c',s'} \begin{cases} 1 & \text{if } c = c' \\ 1 + t_{s'} & \text{otherwise} \end{cases} \tag{2.10}$$

where $t_{s'} > 0$. Also note that $p(\phi, \phi')$ is the price charged by firm ϕ' when selling to firm ϕ.

Firm Pricing and Firm-to-Firm Matching Note that the price elasticity of demand for a firm's variety only varies across sectors, therefore within each sector pair, a firm does not find it optimal to price discriminate. Firms will find it optimal to price discriminate across sectors, but not within a given sector they are selling to. Given this and the demand function for each firm's variety each selling firm charges the common CES markup over their unit cost:

$$p(\phi, \phi') = \mu_{s,s'} \eta(\phi') \tag{2.11}$$

where $\mu_{s,s'} \equiv \sigma_{s,s'}/(\sigma_{s,s'} - 1) > 1$, is the markup over the unit cost of production that is charged by all sector s' firms when selling to sector s. Firm ϕ' similarly charges a markup over unit costs when selling to the representative household employed in any sector or country is given by:

$$p^{HH}(\phi') = \mu_s^{HH}\eta(\phi') \tag{2.12}$$

where $\mu^{HH} \equiv \sigma^{HH}/(\sigma^{HH} - 1)$.

Given the firm's optimal pricing rule, we can now calculate the profit the firm would earn from selling to households in any given country-sector pair.

$$\pi_{s',c'}^{HH}(\phi) = \frac{(\mu^{HH} - 1)}{1+\beta}\frac{I_{s',c'}}{P_{s',c'}}\left[\frac{\alpha_s^{HH}P_{s',c'}}{\mu^{HH}}\right]^{\sigma^{HH}}[\tau_{s,c}^{c'}\eta(\phi)]^{1-\sigma^{HH}} \tag{2.13}$$

Aggregating over household demand from all sectors and countries gives:

$$\pi^{HH}(\phi) = \sum_{s',c'} \pi_{s',c'}^{HH}(\phi) \tag{2.14}$$

Likewise it is also straightforward to calculate the potential profit that firm ϕ' would realize from selling to firm ϕ. Conditional on being able to match with firm ϕ, firm ϕ' would earn a profit of

$$\pi(\phi, \phi') = \frac{(\mu_{s,s'} - 1)[\mu_{s,s'}]^{-\sigma_{s,s'}}}{1+\beta}[\tau_{r',s'}^{r}\eta(\phi')]^{1-\sigma_{s,s'}}x_{s'}(\phi)[P_{s'}(\phi)]^{\sigma_{s,s'}} \tag{2.15}$$

Given the assumption that the selling firm must pay a fixed cost to match with each customer firm they take on as a customer, the selling firm will agree to sell to a customer-firm if the profit of making the sell is greater than the fixed cost of obtaining the customer.

I assume that the relationship fixed cost for each customer obtained by a firm in country-sector pair s, c is equal to $f_{s,c}\xi$, where ξ is a random component. The random component is assumed to be distributed with unit mean according to the cumulative distribution function F_ξ. This fixed cost must be paid in terms of labor which implies the firm must hire an additional $f_{s,r}\xi$ workers for each customer firm they choose to match with, which imposes additional costs. Thus for each match with a customer firms, the selling firm must pay a total of $b_{s,c}f_{s,c}\xi$ in matching costs. Given these assumptions, firm ϕ will find it profitable to sell to firm ϕ' with probability:

$$m(\phi', \phi) = F_\xi\left[\frac{\pi(\phi', \phi)}{b_{s,c}f_{s,c}}\right] \tag{2.16}$$

The total labor employed by a firm to facilitate matching with all other firms can be calculated as:

$$FC(\phi) = f_{s,c}\sum_{c',s'}\int \mathbb{E}_\xi[\xi_{max}(\phi', \phi)]dF_{\phi_\ell}(\phi'_\ell) \tag{2.17}$$

Where $\xi_{max}(\phi', \phi)$ is the highest level of ξ at which firm ϕ would be willing to sell to firm ϕ'. Note that this is given by the following amount:

$$\xi_{max}(\phi', \phi) = \frac{\pi(\phi', \phi)}{b_{s,r}f_{s,r}}$$

$\mathbb{E}_\xi[\xi_{max}(\phi', \phi)]$ can be calculated for any functional form of F_ξ such that the function is defined. In what follows I assume that ξ is distributed according to the Weibull distribution. Solving the model is in no way dependent on this assumption, the model simply requires that F_ξ be a differentiable CDF. The Weibull distribution is advantageous here because it has the ability to assume the characteristics of many different types of distributions. Given the parametric assumption that ξ is distributed according to the Weibull distribution, the partial expectation of ξ_{max} is given by the following:

$$\begin{aligned}\mathbb{E}_\xi[\xi_{max}(\phi', \phi)] &= \int_0^{\xi_{max}(\phi', \phi)} \xi dF_\xi(\xi) \\ &= \int_0^{\frac{\pi(\phi', \phi)}{b_{s,r}f_{s,r}}} \xi\frac{k}{\lambda}(\frac{\xi}{\lambda})^{k-1}e^{-(\xi/\lambda)^k}dF_\xi(\xi)\end{aligned}$$

where k and λ are the shape and scale parameters, respectively, of the Weibull distribution.

2.2 Labor Market

The labor market features frictions such that firms cannot costlessly find workers to hire and must match with workers via a Cobb-Douglass matching function. Firms must match with enough labor to use in production and to additionally pay their fixed costs of matching with other firms. This implies that the fixed cost of matching can be thought of as a communication cost between the customer and supplier or as an upkeep cost of each transactions.[5] Firms post employment vacancies and must pay a cost for each of these vacancies that they post. The cost of posting these employment vacancies endogenously translates into a cost of hiring each worker. The cost of posting a vacancy differs across countries and sectors leading to sector-level comparative advantages across countries. The varying cost of posting vacancies therefore determines sector-level network centrality and the rate at which shocks propagate throughout the economy. Due to the labor-market frictions, after a firm has matched with enough employees to fulfill its total labor demand, the two sides bargain over the surplus generated by their match. Thus the workers are paid a share of variable profits from the firm they work for.

Within each country there is a representative household, with labor supply denoted by $\bar{L}_c$. The household allocates its labor across sectors to maximizes its income. For there to be employment in each sector in the economy it must be the case that within each country expected wages are equalized. Let the *worker-firm matching function* be defined as a homogeneous of degree one Cobb-Douglass function:

$$H_{s,c} = m^L_{s,c}(V_{s,c})^{\chi}(L_{s,c})^{1-\chi} \tag{2.18}$$

The weight of vacancies must be a proper weight, $\chi \in (0, 1)$. The efficiency of labor-market matching in each sector country combination is given by $m^L_{s,c} > 0$. The total number of labor-market matches in sector-country s, c is denoted as $H_{s,c}$. Likewise, the number of vacancies posted by firms in sector-country s, c is given by $V_{s,c}$. The number of workers the country c household has chosen to search for employment in sector s is represented by $L_{s,c}$.

[5]Real-world examples of this fixed cost include time spent communicating with or finding customers, customization of the production process, billing expenses etc...

In order to solve for the optimal number of vacancies that each firm posts, it will be useful to recall demand for labor used in production by each firm.

$$\ell(\phi) = \alpha_s^{\sigma_{\ell,s}} \phi_\ell^{\sigma_{\ell,s}} [b_{s,c}]^{-\sigma_{\ell,s}} Y(\phi) \eta(\phi)^{\sigma_{\ell,s}}$$

The firm's demand for labor to be used in production depends on their firm-specific labor augmenting productivity parameter, the cost of posting vacancies, the labor share of production, and a firm specific input-demand shifter, $Y(\phi)\eta(\phi)^{\sigma_{\ell,s}}$. The total amount of labor demanded by each firm is given by the labor the firm uses in production and the labor the firm uses to pay its fixed cost of matching with customer firms:

$$L(\phi) = \ell(\phi) + FC(\phi) \tag{2.19}$$

The probability that a country c worker searches in sector s and finds a match is $H_{s,c}/L_{s,c}$. Due to wage bargaining, the total payroll of workers employed at a firm is a share of profits equal to $\beta/(1+\beta)$. Therefore the expected wage of searching in sector s is given by:

$$\mathbb{E}(w_{s,c}) = \frac{H_{s,c}}{L_{s,c}} \frac{\beta}{1+\beta} \int \frac{\pi(\phi) - FC(\phi)}{L(\phi)} dF_{\phi_\ell}(\phi_\ell | s, c) \tag{2.20}$$

for there to be employment in all sectors it must be the case that for each sector pair s and s' in all countries, it must be the case that:

$$\mathbb{E}(w_{s,c}) = \mathbb{E}(w_{s',c}) \tag{2.21}$$

Finally, the total number of workers searching across sectors in each country must sum up to equal the total population of workers in the country:

$$\bar{L}_c = \sum_s L_{s,c} \tag{2.22}$$

To describe the firm's employment problem, the total number of vacancies posted in each sector-country is given by summing across the number of vacancies posted by each firm:

$$V_{s,c} = \int V(\phi) dF_{\phi_\ell}(\phi_\ell | s, c) \tag{2.23}$$

where $V(\phi)$ is the total number of vacancies posted by each firm. The cost of hiring each worker, $b_{s,c}$, for the firm equals the total that the firm spends on posting vacancies to hire each worker divided by the number of workers that the firm hires.

$$b_{s,c} = \nu_{s,c} \frac{V(\phi)}{L(\phi)} \tag{2.24}$$

where the parameter $\nu_{s,c}$ gives the cost of posting each employment vacancy.

Finally, the labor market must clear. That is the number of labor-market matches in a given sector-country must equal total employment by firms in the same sector-country:

$$H_{s,c} = \int L(\phi) dF_{\phi_\ell}(\phi_\ell | s, c) \tag{2.25}$$

2.3 Competitive Equilibrium

Solving the model requires two aggregate conditions. First, the goods market must clear and second, the income of households in each country-sector combination must equal total wages paid by firms in the respective country-sector combination.

Firms sell their output to households and any other firm in any sector country that they agree to match with (by paying their fixed cost of matching). Thus the total output of the firm must equal their total sales:

$$Y(\phi) = x^H(\phi) + \sum_{s'r'} \int m(\phi', \phi) x(\phi', \phi) dF_{\phi_\ell}(\phi'_\ell) \tag{2.26}$$

Solving this equation (equation 2.1) and the unit cost equation (equation 2.5) gives the solution to each firm's problem as each firm-specific variable in the model can be written in terms of the two variables these equations define, given the other sector-country specific variables.

The total income of households in each country-sector pair is calculated by aggregating over the total payments to workers by firms. Since the workers receive a share of profits equal to $\beta/(1+\beta)$ this implies that total income in each sector-country must be equal to a share of total sector-country profits. The total variable profit for each firm is the sum of the variable profit from selling to households and each of its customers:

$$\pi(\phi) = \pi_H(\phi) + \sum_{s',c'} \int m(\phi', \phi)\pi(\phi', \phi) dF_{\phi_\ell}(\phi'_\ell) \tag{2.27}$$

This implies that the total income of workers in sector-country s, c is given by:

$$I_{s,c} = \frac{\beta}{1+\beta} \int \pi(\phi) - FC(\phi) dF_{\phi_\ell}(\phi_\ell | s, c) \tag{2.28}$$

Having closed the model I now define a competitive equilibrium of the model:

Definition 2.1. Given a set of parameters and a firm-level distribution of labor augmenting productivities, a *competitive equilibrium* consists of a set of variables that maps from the binary Cartesian power of the set of all firms,

$$\{x(\phi, \phi'), p(\phi, \phi'), \pi(\phi, \phi'), m(\phi, \phi')\}_{\forall \phi, \phi'}$$

, a set of variables that maps from the Cartesian product of the set of all firms and the set of country-sector pairs, $\{x^{HH}_{s',c'}(\phi), p^{HH}(\phi), \pi^{HH}_{s',c'}(\phi)\}^{c' \in \{H,F\}}_{\forall \phi, s'}$ a set of variables that maps from the Cartesian product of the set of all firms and the set of all sectors: $\{P_{s'}(\phi), x_{s'}(\phi)\}_{\forall \phi, s'}$, a set of variables that maps from the set of all firms,

$$\{x^H(\phi), X(\phi), \ell(\phi), x(\phi), \eta(\phi), P(\phi), \pi_H(\phi), FC(\phi), L(\phi), V(\phi), \pi(\phi)\}_{\forall \phi}$$

and a set of variables that maps from the set of all sectors and countries,

$$\{L_{s,c}, H_{s,c}, V_{s,c}, I_{s,c}, b_{s,c}, P^H_{s,c}\}^{c\in\{H,F\}}_{\forall\, s}$$

such that equations 2.1 through 2.28 are satisfied for all firms and sectors in H and F.

2.3.1 The Role of Labor Market Frictions in Production Network Formation

Relative labor market frictions across sectors within each country, $\nu_{s,c}/\nu s', c$, are a determinant of comparative advantage. The relative efficiency of each sector at matching job searchers with firms will determine the distribution of job searchers across sectors. From equations 2.25 and 2.24 a sector with a large labor supply will in general have a a lower cost of hiring workers ($b_{s,c}$).

When the cost of hiring workers changes there are 3 effects on the extensive margin of trade between firms:

1. *Output Effect*: change in labor demand for production by each firm, $\ell(\phi)$. This effect appears in equation 2.4. Since labor and aggregate inputs are related in the production process, changes in the cost of hiring labor will lead to changes in the demand for aggregate inputs by the firm. This change in demand for inputs will lead to a change in the firm's ability to attract suppliers, since the profit from supplying to the firm will have changed. For example, if the firm's aggregate demand for inputs increases then the firm will be more profitable for supplying firms to match with, see 2.15. This would lead to an increase in the number supplier-linkages for the firm.

2. *Efficiency Effect*: change in unit costs of production for the firm, $\eta(\phi)$. This effect appears in equation 2.5. If the cost of hiring a worker falls this leads to a lower unit cost of production, which ultimately leads to an increase in a firm's set of suppliers. This drives changes in the extensive margin of through the intensive margin of trade between the firms. If the selling firm's unit cost falls then the potential profit of the match increases for the customer firm, again see equation 2.15. This would lead to an increase in the cardinality of the firm's set of

customer-firms.

3. *Direct Network Effect*: change in the matching function for the firm, $m(\phi', \phi)$ due to a change in the firm's hiring costs. This effect is made evident by equation 2.16. If a firm's cost of hiring one worker decreases then the firm will be able to match with more customer firms, using the same amount of labor. In contrast to the previous two effects, this effect occurs only along the extensive margin of firm-to-firm trade.

To illustrate each of these effects I differentiate equations 2.16 and 2.15:

$$\frac{\partial m(\phi', \phi)}{\partial b_{s,c}} = \frac{1}{f_{s,c} b_{s,c}} \left(\frac{\partial \pi(\phi', \phi)}{\partial b_{s,c}} \right) - \underbrace{\frac{\pi(\phi', \phi)}{f_{s,c}(b_{s,c})^2}}_{\substack{\text{Direct Network} \\ \text{Effect}}} \tag{2.29}$$

$$\frac{\partial \pi(\phi', \phi)}{\partial b_{s,c}} = A_{s,c}^{s',c'} \left(\underbrace{[\eta(\phi)]^{1-\sigma_{s',s}} \frac{\partial x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}}}{\partial b_{s,c}}}_{\text{Output Effect}} + \underbrace{\frac{\partial [\eta(\phi)]^{1-\sigma_{s',s}}}{\partial b_{s,c}} x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}}}_{\text{Efficiency Effect}} \right) \tag{2.30}$$

where

$$A_{s,c}^{s',c'} \equiv \frac{(\mu_{s',s} - 1)[\mu_{s',s}]^{-\sigma_{s',s}} [\tau_{c,s}^{c'}]^{1-\sigma_{s',s}}}{1 + \beta}$$

The direct-network-effect comes from the standard assumption that the fixed cost of matching must be paid in terms of labor. It is a negative effect, increasing hiring costs leads to less downstream matches.

The efficiency-effect reflects the fact that firms use labor in the production process. This is also a negative effect, increasing the cost to hire one more worker causes firm's unit costs to rise, and since there are constant markups, the price charged by the firm rises. This leads to less downstream matches, in addition to lowering the intensive margin of each potential downstream relationship.

The output effect is less obvious than the efficiency- or direct-network-effect. Suppose $b_{s,H}$ were to rise, how would this effect a sector-s'-firm's demand-shifter for sector-s inputs $(x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}})$? When $b_{s,H}$ increases this leads to workers move out of sector-s and into sector-s'. The greater labor supply available to sector-s' will change the demand for inputs by sector-s'-firms.

The sign of the output effect depends on the elasticity of substitution between aggregate inputs and labor, and whether or not the firm-to-firm match is occurring across or within sectors. If aggregate inputs are substitutes for labor, then the output effect is negative across sectors. That is, increasing $b_{s,H}$ will decrease the demand for aggregate inputs by sector-s'-firms, including sector-s-firms. This implies that the sector-s'-firm's demand-shifter for sector-s inputs would decrease in response to an increase in $b_{s,H}$. Likewise if aggregate inputs are complements for labor ($\sigma_{\ell,s} < 1$), then the output effect is positive across sectors.

Proposition 2.1. If $\sigma_{\ell,s} > 1$, and $s' \neq s$, then $\partial x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}}/\partial b_{s,c} < 0$ and the output effect is negative. And if $\sigma_{\ell,s} < 1$, and $s' \neq s$, then $\partial x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}}/\partial b_{s,c} > 0$ and the output effect is positive.

When the match occurs within a sector, the sign of the output-effect is the opposite of what it would be if the match occurred across sectors. If aggregate inputs are substitutes for labor, then as $b_{s,H}$ increases and workers move out of sector-s, a sector-s-firm's demand-shifter for all inputs (including sector-s inputs) would increase. Therefore if aggregate inputs are substitutes then the output effect is positive within a sector. Likewise, if aggregate inputs are complements then the output effect will be negative across sectors.

Proposition 2.2. If $\sigma_{\ell,s} > 1$, and $s' = s$, then $\partial x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}}/\partial b_{s,c} > 0$ and the output effect is positive. And if $\sigma_{\ell,s} < 1$, and $s' = s$, then $\partial x_s(\phi')[P_s(\phi')]^{\sigma_{s',s}}/\partial b_{s,c} < 0$ and the output effect is negative.

Consider the role of labor market frictions in the number of upstream and downstream linkages for a given firm. The output effect matters for a firm's set of suppliers while the efficiency effect and the direct-network effect matters for a firm's set of customer-firms.

The cardinality of the set of customers for firm-ϕ is given by:

$$|C(\phi)| = \sum_{s',c'} \int m(\phi', \phi) dF_{\phi}(\phi') \tag{2.31}$$

while the cardinality of the set of suppliers for firm-ϕ is given by:

$$|S(\phi)| = \sum_{s',c'} \int m(\phi, \phi') dF_{\phi}(\phi') \tag{2.32}$$

I will aggregate these variables across firms within each sector and treat these as sector level measures of production-network "centrality", denoted by $|S_{s,c}|$ and $|C_{s,c}|$.

An increase in the relative cost of posting an employment vacancy, $\nu_{1,H}/\nu_{2,H}$, leads to $b_{1,H}$ increasing and a decrease in $b_{2,H}$. When inputs and labor are substitutes, this leads to identical increases in $|C_{1,H}|$ and $|C_{2,H}|$ while $|S_{1,H}|$ will fall and $|S_{2,H}|$ will rise. This is due to the aggregation of the output effect, efficiency effect, and the direct network effect across all firms and sectors.

The direct network effect, output effect, and efficiency effect summarize how a change in hiring costs lead to changes in the extensive margin of trade between firms. When there is a change in one sector's labor market frictions, this changes the relative labor market frictions and induces workers to move from one sector and into the other sector. This effects the ability of firms to form firm-to-firm linkages that differs across sectors. This stands in contrast to an increase in the parameter that governs the sector-level average matching ability, $f_{s,c}$, which leads to an increase in the matching ability of all sectors.

2.3.2 The Effect of Network Frictions on Employment

When there is an decrease in sector-level network frictions there is only an effect along the extensive margin of trade. When there is a decrease in $f_{s,c}$ for sector s, profits (and thus wages) increase in all sectors of the economy. However this increase will be smaller in sector-s', causing a reallocation of workers across sectors. This mechanism is similar to that in Helpman and Itskhoki (2010), when there is a reduction in the fixed cost of of exporting to a foreign country, however it differs in two critical ways.

First it differs by the structure of trade, in Helpman and Itskhoki (2010) once a firm pays a fixed cost to export, the firm can link with all customers in the foreign economy. In their framework an adjustment of the exporting friction will lead to a less smooth response in the expected wage across sectors than in the model presented in this paper. In the model presented in this paper, the network friction is per firm match. Adjustment of the firm-matching friction will lead to smoother labor market adjustment than if the fixed cost was paid to become an exporter, granting the firm access all firms in another market.

Secondly there are spillovers due to the network adjustments that occur across sectors. When $f_{s,c}$ increases sector-s-firms find it more profitable to match with firms across all sectors. This effect occurs due to two competing factors that can be seen from revisiting the labor demand equation 2.4.

$$\ell(\phi) = \alpha_s^{\sigma_{\ell,s}} \phi_\ell^{\sigma_{\ell,s}} [b_{s,c}]^{-\sigma_{\ell,s}} Y(\phi) \eta(\phi)^{\sigma_{\ell,s}}$$

The first mechanism is a scale effect, that is captured by changes in $Y(\phi)$. As firms are able to make more sales, labor demand increases due to increases in total output of the firm. The second effect is a gains from suppliers effect and occurs through changes in each firm's unit cost, $\eta(\phi)$. As firms gain access to more suppliers, $\eta(\phi)$ falls due to the gains from variety in production.

The increase in $f_{s,c}$ leads to a relative increase in labor demanded for production by all firms in sector-s. This is because the scale effect dominates the gains from suppliers. An increase in $f_{s,c}$ increases the firm's mass of suppliers and customers for firms in all sectors. However the increase in $f_{s,c}$ will lead to a greater increase in the mass of customers for sector-s firms, relative to sector-s'-firms. Therefore, when $f_{s,c}$ increases this leads to a relative increase in labor demand for use in production in sector-s.

Wage Inequality and Network Efficiency This model can also be used to study within sector wage inequality. Workers are matched to firms randomly within a given sector. The distribution of firm-level labor augmenting productivities, and the wage bargaining leads to a distribution of wages within each sector. To understand how the firm-level labor augmenting productivity (ϕ_ℓ) drives changes in wages, note that the unit-cost equation, 2.5, is decreasing in ϕ_ℓ. This implies that firms with a high level of ϕ_ℓ will find it profitable to match with more customer-firms, than firms with a lower level of ϕ_ℓ. Since firms only match when it is profitable, more matches with customer-firms implies greater profits.

When the matching cost, $f_{s,c}$, increases, firms with lower levels of the labor augmenting productivity, ϕ_ℓ, will now find it profitable to match with customers. This implies average profits and wages will increase for all firms, but the greatest increase will be realized by firms in the left half of the distribution of firms.

The exact response of wage inequality depends on the assumptions placed on the distribution of the firm matching shocks. Firms with a high level of ϕ_ℓ will find it beneficial to sale to a large customer base, independent of $f_{s,c}$. These firms have a low cost of production, so their firm-relationships are not as sensitive to the parameter $f_{s,c}$. Relationships with customer-firms for suppliers with lower levels of ϕ_ℓ are more sensitive to the parameter $f_{s,c}$ and the distribution of the random component of matching, ξ.

2.4 Globalization and Tariffs

I will model Globalization as a decrease in international shipping costs and international matching costs. A change in shipping costs will lead to a different result from a change in matching costs. Therefore I separate these effects in my discussion of globalization.

2.4.1 A Reduction in Firm Matching Costs

When there is a symmetric change in international matching costs between countries, it is easier for all firms in Home to find more customer-firms and suppliers. However it is possible for one sector in the home country to become relatively more “central" in response to a symmetric reduction in these international matching costs. This depends on two factors: the sector-level comparative advantage for countries, determined by their labor market efficiency, and the concavity of the matching function for the average firms in each sector.

First, it must be the case that the Home country has a comparative advantage in one of the sectors. Helpman and Itskhoki (2010) show how relative labor market frictions determine comparative advantage for countries. They show that if sector-s in Home has a more efficient labor market, relative to sector-s', than sector-s in Foreign, then Home will be a net exporter of sector-s varieties. In my model this feature still holds. However, combining it with the concavity of the matching function will yield predictions about how employment and unemployment will change in response to a reduction in firm-matching costs.

When the matching function for the average firm pairs in each sector is concave a change in the parameters or variables that determine the likelihood of a match will be less than if the matching function were convex. That is, when the matching function is convex firm to firm matches a more sensitive.

Proposition 2.3. Suppose Home has a comparative advantage in sector-s, if there is a reduction in international firm matching frictions:

1. If the firm-matching function is convex, Home will increase its net-exports of sector-s varieties and Foreign will increase its net-exports of sector-s' varieties.

2. If the firm matching function is concave, Home will decrease its net-exports of sector-s varieties and Foreign will decrease their net exports of sector-s'

The intuition of this result is as follows. If Home has a comparative advantage in sector-s varieties, then the average firm in sector-s has more matches with customer-firms than its sector-s' counterparts. If the matching function is convex and the fixed cost of firm matches falls, then sector-s primes will find it more profitable to match with even more foreign customers. The average sector-s' firm will also find it more profitable to match with more foreign customers, but this increase will be less than the increase in sector-s.

These gains in exports will cause workers to reallocate across sectors in response to a global reduction in international firm-matching costs. If the firm-matching function is concave, globalization as represented by a decrease in international matching costs will lead to a reallocation of labor out the net-exporting sector and into the net-importing sector.

This reduction in firm-matching costs will also have effects on wage inequality within and across each sector. Each sector will benefit from more matches, allowing more firms to export and thus reducing wage inequality within each sector. Again this effect depends on sector level comparative advantage and the concavity of the average firm's firm-matching function.

The mechanism driving this is the same as the mechanism laid out in Proposition 2.3. If the matching function is concave, globalization as represented by a decrease in international matching costs will lead to an increase in relative average wages in the net-importing sector. This is due to convergence in profits across sectors. Thus if the firm-matching function is concave, a reduction in network frictions will lead to a reduction in wage inequality across sectors.

However, if the firm-matching function is convex for the average firm in each sector, then a decrease in the firm matching frictions leads to greater gains in profits for the net-exporting industry. This implies that there is an increase in wage inequality across sectors if the firm-matching function is convex.

Within sector wage inequality will depend on the concavity of the matching function. Sector pay will become more unequal as firm profits diverge, due to wage-bargaining that occurs over firm profits. If the firm-matching function is concave, firm profits will increase by more for less productive firms in response to the reduction in the firm matching fixed costs. This implies that a fall in firm matching fixed costs will lead to a reduction in within sector wage inequality. Likewise if matching costs are convex, then within sector wage inequality will rise in response to a fall in firm matching costs.

2.4.2 A Reduction in Iceberg Trade Costs

I now consider the effects of globalization as represented by a symmetric reduction in international trade costs. Changes in iceberg trade costs will yield different predictions than a change in international matching frictions. This is due to the fact that international matching frictions will directly affect the extensive margin of trade, and indirectly affect the intensive margin of trade. A change in iceberg trade costs will directly impact both the intensive margin of trade and the extensive margin of trade.

The set of results for sector-level outcomes in each country will be similar for both a symmetric reduction in iceberg and a symmetric reduction in matching frictions. The results for iceberg trade costs will simply be amplified relative to changes that would occur from a reduction in network costs. This is due adjustments that occur along the intensive margin of trade.

Revisiting equation 2.15, a reduction in iceberg trade costs will directly impact $\pi(\phi, \phi')$ for all firm-pairs where $c \neq c'$. This implies not only that the number of matches will increase for firms, but also that the average profit per match will increase for all firms.

Therefore it follows that:

Proposition 2.4. Suppose Home has a comparative advantage in sector-s, if there is a reduction in international iceberg trade costs, and if the firm matching function is...:

1. Convex, Home will increase its net-exports of sector-s varieties and Foreign will increase its net-exports of sector-s' varieties.

2. Concave, Home will decrease its net-exports of sector-s varieties and Foreign will decrease their net exports of sector-s'

However since there is a direct adjustment along the extensive margin in 2.4, in addition to the adjustment described in proposition 2.3, the following holds:

Proposition 2.5. The magnitude of the changes in net-exports described in 2.4 is greater than the magnitude of change in net-exports described in 2.3, for a identical percentage change in the profits from matching relative to the cost of matching for the average firm pair, $\overline{\pi(\phi, \phi')/f_{s',c'}b_{s',c'}}$. [6]

This says that a reduction in iceberg trade costs will have larger implications for the distribution of economic activity across sectors, relative to a similar reduction in firm-matching frictions.

A reduction in international matching costs and iceberg trade costs will lead to efficiency gains in an increase in average real wages for employees in all sectors. However the effects on the

[6] Where $\overline{x}$ denotes the average across all firm pairs for variable x.

distribution of wages will differ within and across sectors, depending on the shape of the firm-matching function.

2.4.3 A Protectionist Tariff

A protectionist tariff will reduce the number of potential suppliers to all firms in Home and lead to a reallocation of labor across sectors in Home and Foreign. The effects of a protectionist tariff will differ from a change in symmetric iceberg trade costs due to the “distortion" effect created by the reallocation of labor across sectors in Foreign. A protectionist tariff in this setting can lead to an increase in real income in Home.

When Home imposes a tariff on imports from sector-s, firms located in Home will lose access to potential suppliers. This causes the unit cost of firms in Home to increase. From equation 2.4, as unit costs ($\eta(\phi)$) increase labor demanded will increase as well. This effect will lead firms to post more employment vacancies in an effort to hire more labor for use in production.

Each firm in Home will demand more aggregate inputs as well. This effect is driven by the increase in unit costs, $\eta(\phi)$, from the decrease in suppliers. This is can also be seen in equation 2.4. This implies that the loss in suppliers leads to an increase in labor demand in Home, and a “thickening" of the domestic production-network. Firms will demand more inputs from one another and trade more within the Home country.

At the same time, when Home places a tariff on imports from a specific sector in Foreign, Foreign job-searchers move out of the sector that is facing the tariff and into the sector that is not. This leads to a reduction in unit costs in the non-taxed sector in foreign, This makes the firms in the non-tariff sector more viable as exporters, and offsets the loss in suppliers from the tariff, leading to a slight increase in demand for labor and inputs.

This reallocation of Foreign labor also offsets losses in the household price index. Foreign firms in the non-taxed sector experience a reduction in unit costs, due to a reduction in the cost of hiring workers. This savings is passed on to domestic firms and households. These effects are enough such that a protectionist tariff in one sector can lead to an increase in nominal and real income. Furthermore it can lead to a reduction in the household price index.[7]

3 Model Simulations

In this section I present the results from several simulations of the model. I first lay out the assumptions I place on parameters in the model. I then lay out the set of counterfactual experiments that I simulate for the model. I simulate a relative increase in the efficiency of the sector-1 labor market Home. I then simulate a relative increase in the average production network efficiency for sector-1 firms in Home. I use these simulations to further discuss the propositions outlined in section 2.3.1. To simulate the effect of globalization I simulate a reduction in firm average production network frictions. Finally I simulate the effects of a protectionist tariff for sector-1 in the Home country.

3.1 Simulation Setup

I assume there are two countries, Home and Foreign, each consisting of two sectors, sector-1 and sector-2. I take the price charged by the sector-2 firm with the lowest productivity draw (ϕ_ℓ) in free trade to be the numeraire of the model. I simulate the model using parameters guided by calibrations from Lim (2018) and parameter choices made by Helpman and Itskhoki (2010).

I assume that the firm-level distribution of labor-augmenting productivities is distributed according to a log-normal distribution. I assume the distribution has mean and standard deviation given by μ_{ϕ_ℓ} and σ_{ϕ_ℓ} respectively. I follow Lim (2018) by assuming that the random component of the firm-matching cost is distributed according to the Weibull distribution with shape and scale parameters given by λ and k respectively.

In order simplify the model, I remove one dimension of non-linearities in the model by assuming that the elasticity of production is identical at all stages of the firm production function.[8]

[7] This effect is a long run effect, if workers are not able to move across sectors in the foreign sector (as is typically the case in the short run) then there will be an increase in the domestic price index. Furthermore, this model does not take into account retaliatory tariffs which would erase any gains from the protectionist tariff.

[8] Huneeus (2018) and Baqaee and Farhi (2017) explore the implications of these non-linearities

Parameter	Description	Value
μ_{ϕ_ℓ}	Mean of Log-Normal distribution of firm productivities	0
σ_{ϕ_ℓ}	Standard Deviation of Log-Normal distribution of firm productivities	1
λ	Shape parameter for distribution of the random component of firm-matching costs (ξ)	1
k	Scale parameter for distribution of the random component of firm-matching costs (ξ)	1
α_s^{HH}	Sector-level consumption suitability parameter	0.25
σ^{HH}	Household elasticity of substitution	3.1
α_s	Production function weight of aggregate inputs	0.5
$\sigma_{\ell,s}$	elasticity of substitution between inputs and labor	3.1
β	Worker's bargaining power	0.5
σ_s	Firm's elasticity of substitution between sectors	3.1
$\alpha_{s,s'}$	Input suitability of sector-s' inputs for use in sector-s production.	0.25
$\sigma_{s,s'}$	Elasticity of substitution for sector-s' varieties for use in production by sector-s firms	0.25
$m_{s,c}^L$	Scale of firm-worker matching function	0.95
χ	Weight of employment vacancies in firm-worker matching function	0.25
$\bar{L}_c$	Total mass of workers in country-c	1

Table 1: Parameters Used in Simulation

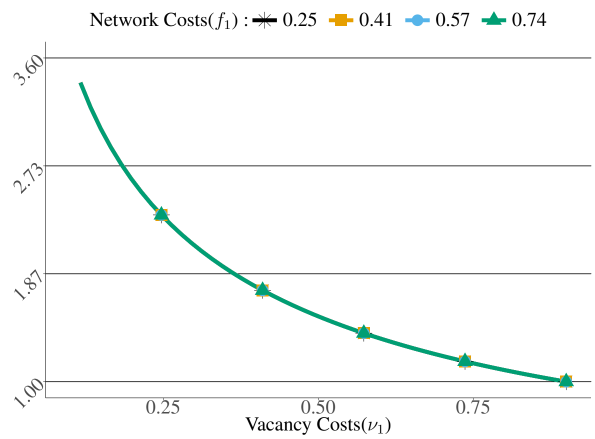
Another feature of note is that the input suitability parameters do not vary by sector pairs. I will explore this feature in a future paper when the model is extended to a dynamic framework and estimated using firm-level data.

Absent from table are the costs of posting an employment vacancy ($\nu_{s,c}$), and the average value of the firm-matching parameter ($f_{s,c}$). These parameters will take on multiple values through the course of the simulations. I simulate the model multiple times where each of these parameters takes a value over the interval $(0, 1)$. These simulations will be discussed in the following sections.

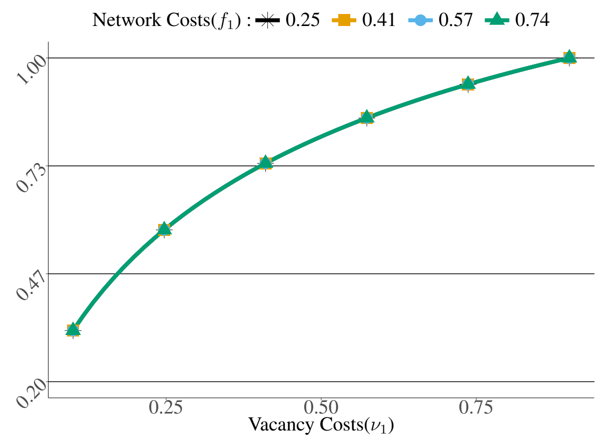
3.2 Relative Changes in Labor Market Efficiency

In this section I simulate an change in the relative labor market efficiency to show how this leads to a reallocation of labor across sectors. I then show how this effect leads to changes in firm-linkages.

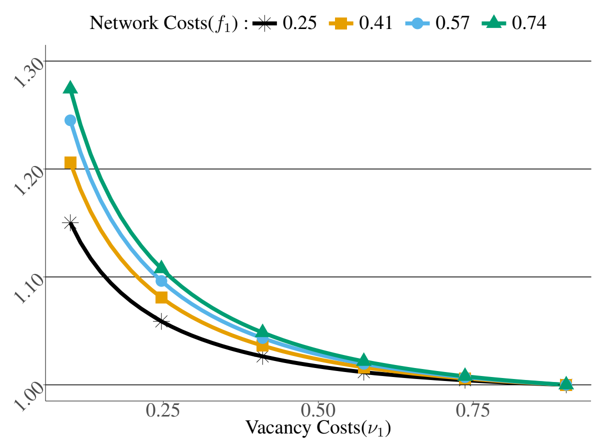
I simulate an change in the cost of posting an employment vacancy for sector-1 in Home ($\nu_{1,H}$) over the interval $(0, 1)$. I hold all other sectors costs of positing an employment vacancy equal to $1/2$. When $\nu_{1,H} < 1/2$, this implies that Home has a comparative advantage in sector 1. As it becomes relatively more expensive for firms to post an employment vacancy in sector 1, job-searchers in sector-1 will move into sector-2.



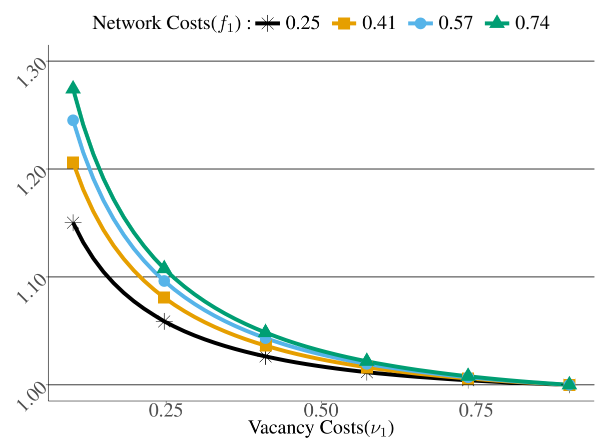
(a) Sector 1 Average Mass of Supplier-Linkages



(b) Sector 2 Average Mass of Supplier-Linkages



(c) Sector 1 Average Mass of Customer-Linkages

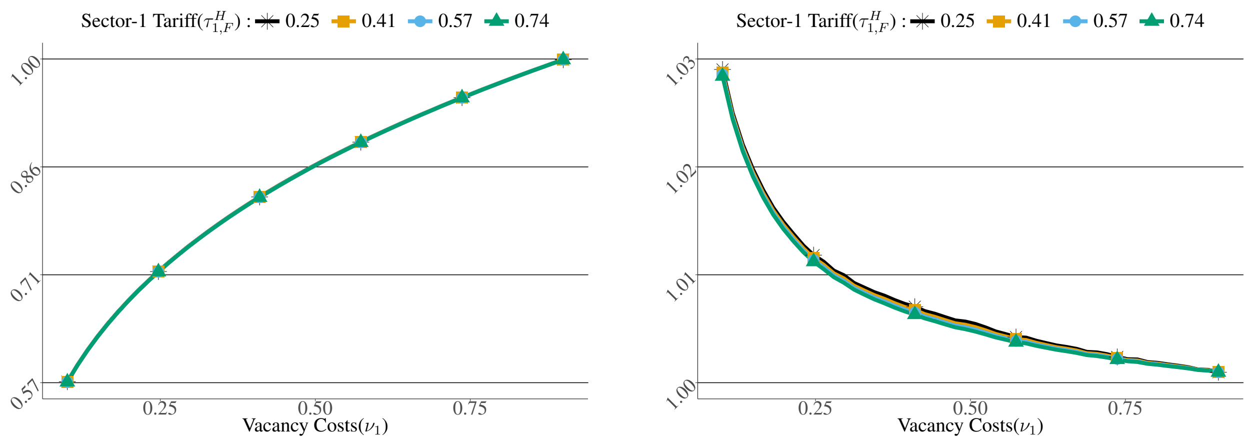


(d) Sector 2 Average Mass of Customer-Linkages

Figure 3.1: Relative Labor Market Frictions and the Production-Network

Figure 3.1 presents the results of the simulated increase in $\nu_{1,H}$. Figure 3.1 shows the mass of customer- and supplier-linkages, for the average firm in each sector. As the cost of posting an employment vacancy increases for sector-1, the average firm's mass of customer-linkages decreases for both sectors. However the average firm's mass of supplier-linkages is decreasing for sector 1, and increases for sector-2.

As the cost of posting a vacancy increases, workers move out of sector-1 and reallocate into sector-2 due to a decreasing probability of finding a match in sector-1. The sector-1 cost of hiring a worker in Home, $b_{1,H}$, increases and the cost of hiring a worker in sector-2 in Home, $b_{2,H}$ falls as ν_1 increases.[9] This is confirmed in figure 3.2. When this occurs, the *direct network effect* leads to a decrease in the mass of customer-linkages for sector-1 firms, and an increase for sector-2 firms. Adding to this, the *efficiency effect* predicts that unit costs will fall in sector-2 and increase in sector-1, further increasing the mass of customers-linkages for sector-2, and depressing it for sector-1.



(a) Sector 1 Cost of Hiring (b) Sector 2 Cost of Hiring

Figure 3.2: Cost of Posting a Vacancy (ν) and the Cost of Hiring (b)

This would suggest that the mass of customer-linkages should increase for sector-2 firms, and decrease for sector-1. However, labor and inputs are assumed to be substitutes, since $\sigma_{\ell,s} > 1$. When the price of labor rises in sector-1, $b_{s,H}$, sector-1 firms increase their demand for aggregate inputs from all sectors. Likewise, as the price of labor falls in sector-2, firms decrease their demand for aggregate inputs. Proposition 2.1 gives this result.

For sector-2 it is the case that the *output effect* is dominated by the *direct network effect* and the *efficiency effect*, leading to an increase in the average-firm's mass of suppliers. Likewise is sector-1, the *output effect* is dominated by the *direct network effect* and the *efficiency effect*.

[9]The increase in $b_{1,H}$ is larger than the decrease in $b_{2,H}$. This is because $b_{1,H}$ rises due to two factors, the increased cost of posting vacancies, and the lower labor supply in sector-1, as job searchers move from sector-1 into sector-2. In comparison, $b_{2,H}$ only increases due to a greater number of job searchers.

3.3 Labor-Market Frictions and Network Centrality

In this section I simulate the effects of an increase in the firm-network frictions for sector-1 in Home, $f_{1,H}$. This effect differs from a relative increase in $\nu_{1,H}$, because there is only a small reallocation of labor when only $f_{1,H}$ changes, relative to a change in $\nu_{1,H}$.

To understand how this model contributes to the literature on production network formation it is useful to compare figure 3.3b and figure 3.1b. Increasing the average cost of matching for sector-1 firms, decreases the mass of supplier-linkages for sector-2 firms. Compare this to the increase in the mass of supplier-linkages for sector-2 firms, as the Average cost of posting an employment vacancy rises in sector-1. This difference is due to the larger reallocation of labor that occurs in the latter. This reallocation of labor effects firms demand for inputs, and leads to a differential impact on the number of suppliers to each sector.

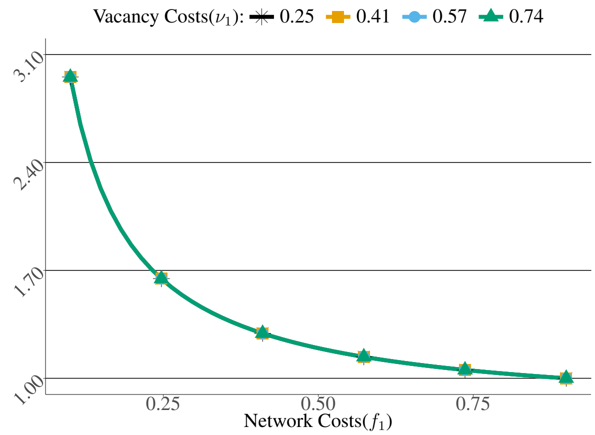
3.4 A Simulated Tariff

In this section I simulate the effects of a tariff for sector-1 by the Home country. The tariff leads to a reduction of cheap inputs for Home and a movement of Foreign labor out of sector-1 and into sector-2 (as shown in figures 3.4a and 3.4b). This reallocation of Foreign labor is consequential for consumers, employment, and real income in Home.

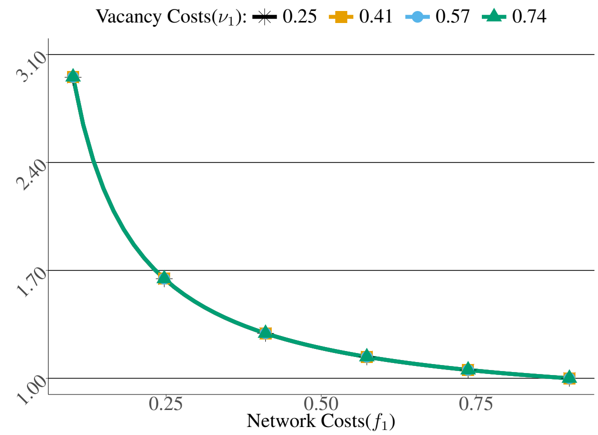
Firms in both sectors in Home form more linkages with customer-firms. This effect is a substitution effect, as firms substitute away from Foreign sector-1 varieties towards others. This effect is a “thickening" of the domestic production-network. This effect is shown in figures 3.4c and 3.4d.

At the same time, labor reallocates out of the Foreign taxed sector and into the sector that is not facing the tariff. Since labor and inputs are substitutes, this induces firms in the foreign taxed sector to increase their demand for inputs. This induces more firms in home to export to Foreign. Note that if labor and inputs were instead complements, the sector not facing the tariff would increase their demand for workers, leading to similar results for Home.

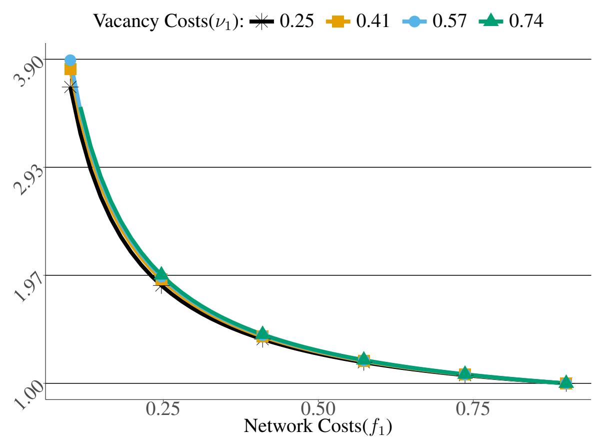
This “thickening" of the domestic production-network, lowers production costs of firms across all sectors in Home, leading to a decrease in the household price index in Home. At the same time



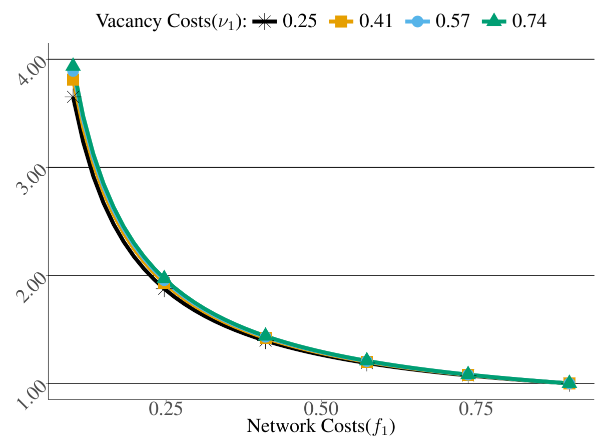
(a) Sector 1 Average Mass of Supplier-Linkages



(b) Sector 2 Average Mass of Supplier-Linkages



(c) Sector 1 Average Mass of Customer-Linkages

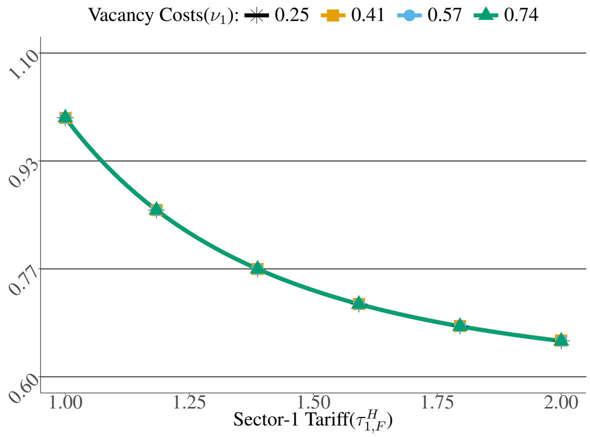


(d) Sector 2 Average Mass of Customer-Linkages

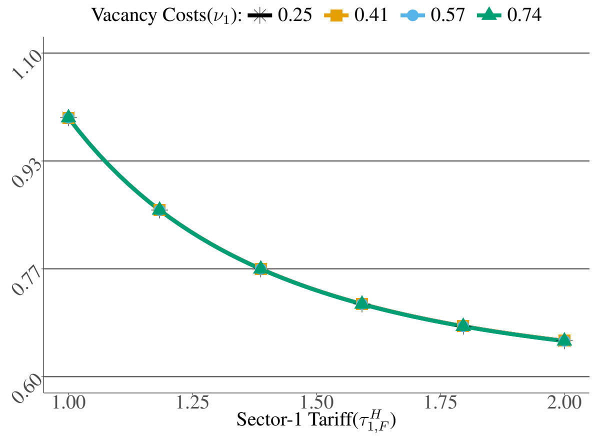
Figure 3.3: Relative Production Network Frictions and the Production-Network

Home-firms export more and make more sales to Home-firms, this leads to an increase in nominal wages. These effects combine to increase real wages in the Home

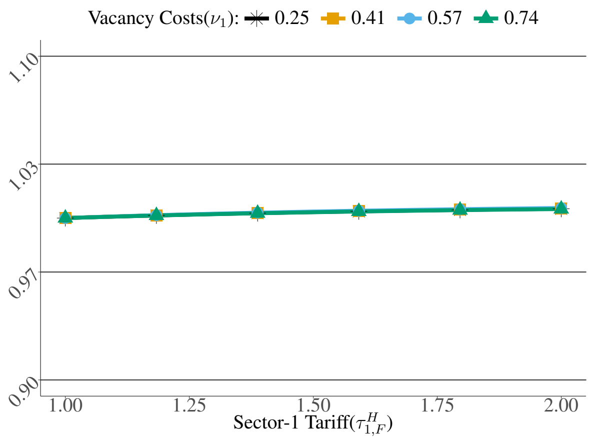
These simulations demonstrate the importance of sectoral reallocation in determining production network centrality, firm profits, and labor market outcomes. Any policy that changes relative wages across sectors will induce a reallocation of workers across sectors. This reallocation of labor will have consequences for how firms choose to form their production-networks.



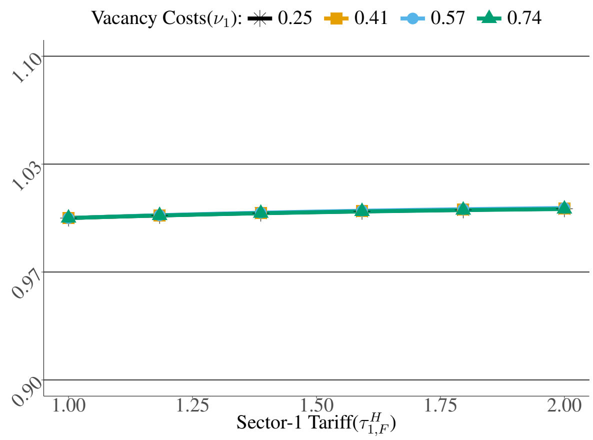
(a) Sector 1 Average Mass of Supplier-Linkages



(b) Sector 2 Average Mass of Supplier-Linkages



(c) Sector 1 Average Mass of Customer-Linkages



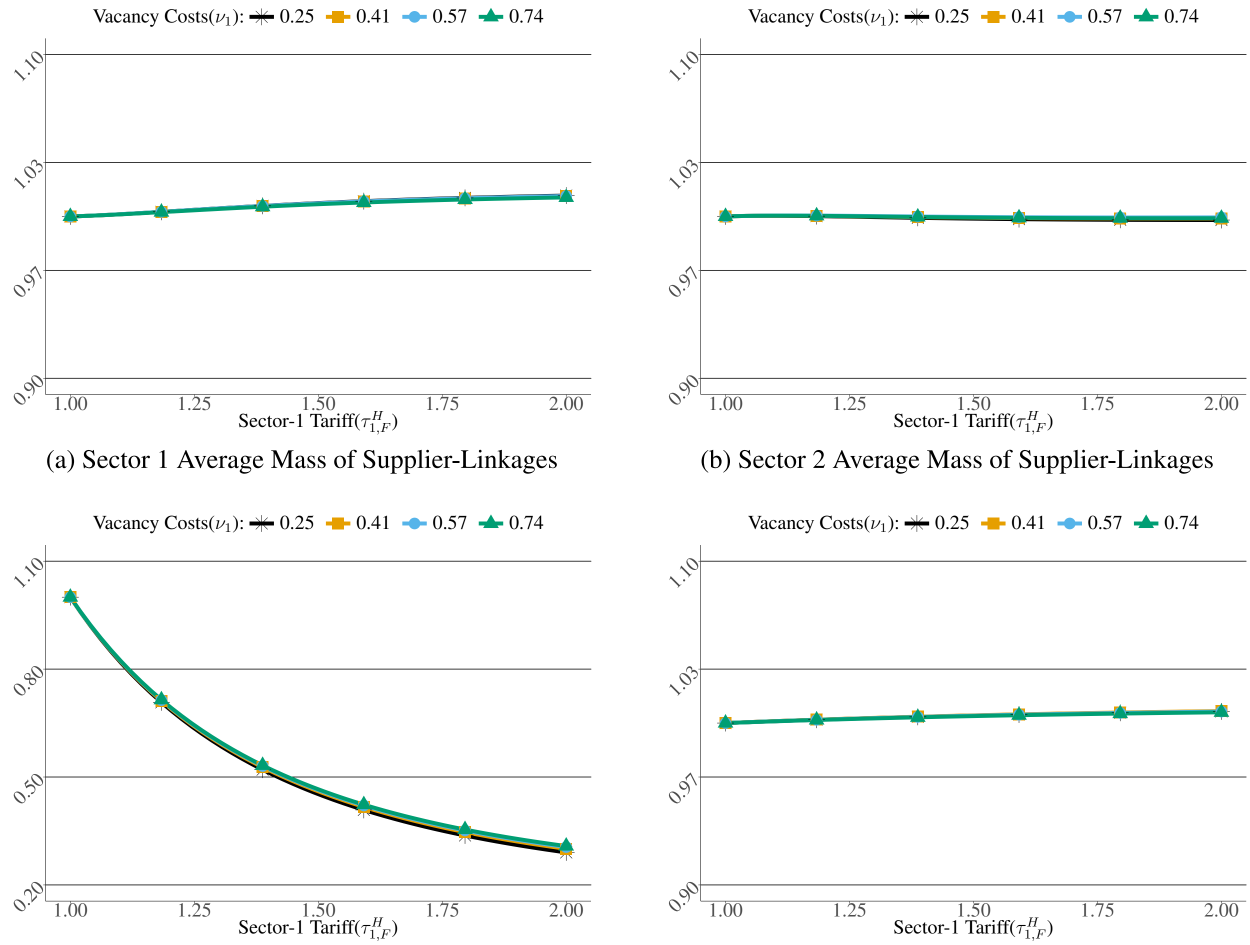
(d) Sector 2 Average Mass of Customer-Linkages

Figure 3.4: A Protectionist Tariff and Home's Production-Network

4 Conclusion

This paper presented a model that demonstrates how endogenous production-network formation and relative labor market frictions interact. The model generates a rich set of predictions on how the production-network changes in response to sector-level relative labor market frictions, and how labor markets change in response to globalization and tariffs.

Future work should be done to empirically study the model in a dynamic setting. Changes in the economy in a setting featuring sticky -employment and -production-networks will lead to meaningful transitions in economic outcome. The larger benefit of extending the model to a dynamic setting is to allow for the structural estimation of the model using firm-level panel data



(a) Sector 1 Average Mass of Supplier-Linkages

(b) Sector 2 Average Mass of Supplier-Linkages

(c) Sector 1 Average Mass of Customer-Linkages

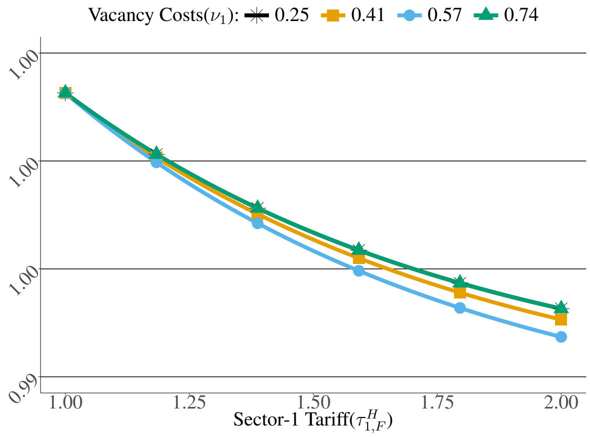
(d) Sector 2 Average Mass of Customer-Linkages

Figure 3.5: A Protectionist Tariff and Foreign's Production-Network

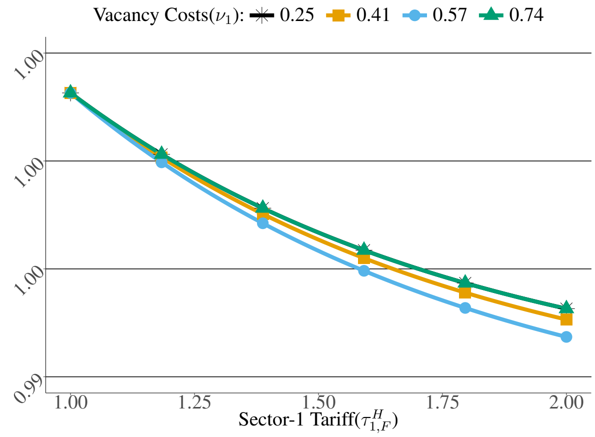
to determine the importance of labor-market frictions in production-network formation, and the importance of production network frictions in determining labor market outcomes.

Understanding how production network formation has changed in response to globalization is critical to understanding how workers have been impacted by trade. This model takes a step in this direction by analyzing the role of labor market institutions and frictions are intertwined with production-network formation. The model demonstrates how worker outcomes change in response to globalization. when the intensive and extensive margins of trade are allowed to change.

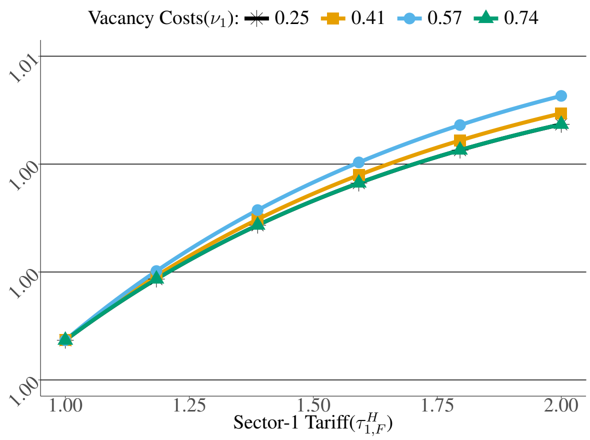
The results on wage inequality suggests that governments can decrease wage inequality in their economy by investing in infrastructure, professional conferences, etc. Investment in anything that



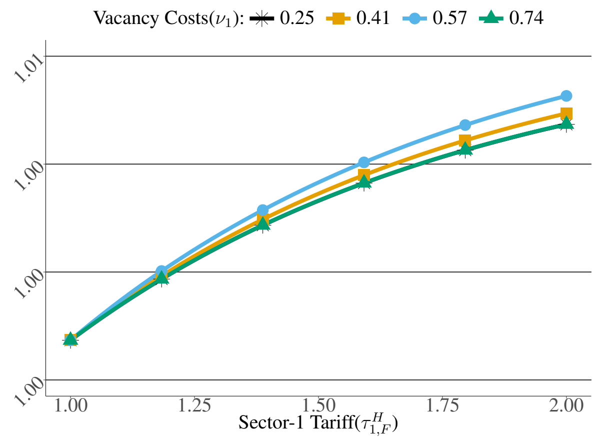
(a) Sector 1 HH-Price Index



(b) Sector 2 HH-Price Index



(c) Sector 1 Real Wages



(d) Sector 2 Real Wages

Figure 3.6: A Protectionist Tariff and Home's Labor Market

reduces the cost of face to face interaction for firms, or simply reduces the costs of doing business for firms can lead to an increase in average wages and a decrease in wage inequality.

The model also demonstrates how the reallocation of workers across sectors leads to long run changes in the production network. When trade policy, or local labor market institutions, change in a way that one sector is made relatively more attractive for workers there will be changes in production-network outcomes.

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